COMMON PRE-BOARD EXAMINATION 2022-23

Subject: MATHEMATICS (041)

Date: Duration: 3 Hours

TOTAL MARKS: 80

General Instructions:

1. This question paper contains **five sections A, B, C, D and E**. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has **18 MCQ's** and **2 Assertion- Reason** based questions of 1 mark each.
3. **Section B** has **5 Very Short Answer(VSA)** type questions of 2 marks each.
4. **Section C** has **6 Short Answer (SA)** type questions of 3 marks each.
5. **Section D** has **4 Long Answer (LA)** type questions of 5 marks each.
6. **Section E** has **3 source based/ case based/ passage based** integrated units of assessment (4 marks each) with sub parts.

SECTION A

(Multiple Choice Questions)

Each question carries 1 mark

Q 1	A is a square matrix such that $A^2 = I$, where I is the identity matrix, then find the value of $(A-I)^3 + (A+I)^3 - 7A$ a)I b) –I **c) A** d) –A	1
Q2	A is a 3x3 invertible matrix, then what will be value of k if $\det(A^{-1}) = (\det A)^k$ a)1 **b) -1** c) 2 d)-2	1
Q3	If $\vec{a}$ is a unit vector such that $(2\vec{x} - 3\vec{a}).(2\vec{x} + 3\vec{a}) = 91$, then find $\|\vec{x}\|$ a)4 **b) 5** c) 6 d) 10	1
Q4	For what value of k, the following function is continuous at x = 0 ? $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2} & if\ x \neq 0 \\ k\ if\ x=0 \end{cases}$ **a)1** b) -1 c) ½ d) – ½	1
Q5	Evaluate $\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cos x}\, dx$ a)π b)$\pi/2$ c) 1 **d) 0**	1
Q6	Write the integrating factor of the differential equation $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$	1

	a) $\tan^{-1} x$ b) e^x **c)**$\boldsymbol{e^{\tan^{-1} x}}$ d) $\frac{1}{1+x^2}$	
Q7	The solution set of the inequality 3x + 4y < 4 is a) an open half plane not containing origin **b) an open half plane containing origin** c) the whole XY [plane not containing the line 3x+4y = 4 d) the whole XY plane containing the origin	1
Q8	Find λ when the projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units a) -5 b) -4 c) 4 **d) 5**	1
Q9	If $\int \left(\frac{x-1}{x^2}\right) e^x = f(x)e^x + \; C, then\ write\ the\ value\ of\ f(x)$ a) x **b) 1/x** c) x^2 d) x-1	1
Q 10	If matrix A = $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ $and\ A^2 = p\ A$, write the value of p. a) 2 b) -2 c) 1 **d) 4**	1
Q11	Let A and B be two events. If P(A) = 0.2, P(B) = 0.4 and P(AUB) = 0.5, then find P(A/B) a) 0 b) ½ **c) ¼** d) 1	
Q12	If A is a square matrix of order 3 such that $\lvert adjA \rvert = 81,$ then find $\lvert A \rvert$. a) ∓ 81 **b)** $\boldsymbol{\mp 9}$ c) ∓ 27 d) ∓ 3	1
Q13	Evaluate $\begin{vmatrix} cos15 & sin15 \\ sin75 & cos75 \end{vmatrix}$ **a) 0** b) 1 c) -1 d) ½	1
Q14	Write the sum of order and degree of the differential equation $\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\}$ = 0 **a) 3** b) 4 c) 5 d) 2	1
Q15	Ans **d) at every point of the line segment joining the points (0.6,1.6) and (3,0).**	1
Q16	What is $\frac{dy}{dx}$ at x = 2 if x – y = k. a) 0 **b) 1** c) -1 d) 2	1
Q17	If $\lvert\vec{a}\rvert = 4\,, \lvert\vec{b}\rvert = 3\ and\ \vec{a}.\vec{b} = 6\sqrt{3},$ then value of $\lvert\vec{a} \times \vec{b}\rvert$. a) 4 b) 3 **c) 6** d)12	1
Q18	If a line makes angles $\alpha, \beta\ and\ \gamma$ with the coordinate axes, write the value of $sin^2\alpha +\ sin^2\beta + sin^2\gamma$ a) 1 **b)2** c) 3 d)0	1
Q19	Assertion (A) : $\sin^{-1}\left(sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$ Reason (R) : $\sin^{-1}(sinx) = x\ if\ x\ \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ a) Both A and R are true and R is the correct explanation of A b) Both A and R are true but R is not the correct explanation of A	

	c) A is true but R is false **d) A is false but R is true**	1
Q20	Assertion (A) : The angle between the straight lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$ is 90°. Reason (R): Skew lines are lines in different planes which are parallel and intersecting. a) Both A and R are true and R is the correct explanation of A b) Both A and R are true but R is not the correct explanation of A **c) A is true but R is false** d) A is false but R is true	1
	SECTION B (2 Marks each)	
Q21	Prove that $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$ $\frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right)$ $\frac{9}{4}\left(\cos^{-1}\frac{1}{3}\right)$ $$\frac{9}{4}\left(\sin^{-1}\frac{2\sqrt{2}}{3}\right)$$ **OR** Let f: N → N be defined by f(x) = $f(x) = \begin{cases} \frac{n+1}{2}, & if\ n\ is\ odd \\ \frac{n}{2}, & if\ n\ is\ even \end{cases}$. For all n ∈ $N,$ state whether f is bijective. Let n = 1, odd, f (1) = 1 Let n = 2, even, f (2) = 1 f(x) is not one – one So f is not bijective	½ ½ 1 ½ ½ ½ ½
Q22	A particle moves along the curve 6y = x^3 + 2. Find the x – coordinate of the points on the curve at which the y – coordinate is changing 8 times as fast as x – coordinate. $6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$ 6 x 8$\frac{dx}{dt}$ = $3x^2\frac{dx}{dt}$ x^2 = 16 x = ± 4	1 1

Q23	If $\vec{a}$ *and* $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ if $\vec{a}-\sqrt{2}\vec{b}$ be a unit vector. $\left\|\vec{a}-\sqrt{2}\vec{b}\right\| = 1$ $\left\|\vec{a}-\sqrt{2}\vec{b}\right\|^2 = 1$ $(\vec{a}-\sqrt{2}\vec{b})(\vec{a}-\sqrt{2}\vec{b}) = 1$ 1 +2 - 2√2a.b = 1 a.b = 1/√2 cosθ = $\frac{a.b}{\|a\|\|b\|}$ = 1/√2 $\theta = 45°$ **OR** If the lines $\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{2z-6}{4}$ *and* $\frac{1-x}{-3\lambda} = \frac{y-1}{1} = \frac{10z-12}{-10}$ are perpendicular to each other, find the value of λ Perpendicular, a.b = 0 -3 x 3λ + 2λ x 1 + 2 x -1 = 0 λ = - 2/7	½ ½ ½ ½ ½ 1 ½
Q24	If $y = logtan\left(\frac{\pi}{4}+\frac{x}{2}\right), then\ show\ that\ \frac{dy}{dx} = secx.$ $\frac{dy}{dx} = \frac{1}{tan\left(\frac{\pi}{4}+\frac{x}{2}\right)} sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\frac{1}{2}$ $= \frac{1}{2sin\left(\frac{\pi}{4}+\frac{x}{2}\right)cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}$ $= \frac{1}{sin\left(\frac{\pi}{2}+x\right)}$ = $\frac{1}{cosx}$ = secx	1 ½ ½
Q25	Let $\vec{a} = \hat{\imath}+4\hat{\jmath}+2\hat{k}$, $\vec{b} = 3\hat{\imath}-2\hat{\jmath}+7\hat{k}$, and $\vec{c} = 2\hat{\imath}-\hat{\jmath}+4\hat{k}$, find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ *and* $\vec{p}.\vec{c} = 18$. P = λ (a x b) a x b = 32i – j -14 k λ(64 +1 -56) = 18 λ = 2	1 ½ ½

SECTION C (3 Marks each)

Q26 Evaluate $\int \frac{x^2}{(x^2+1)(x^2+4)}dx$

Let t = x^2

$$\frac{1}{(t+1)(t+4)} = \frac{A}{(t+1)} + \frac{B}{(t+4)}$$ 1

A = 1/3 and B = -1/3 1

$\int \frac{1/3}{(x^2+1)}$ - $\int \frac{1/3}{(x^2+4)}$ = 1/3 $\tan^{-1} x - \frac{1}{6}\tan^{-1} x/2$ + C 1

Q27 A problem in Mathematics is given to three students whose chances of solving it are ½, 1/3 and ¼. If all of them try to solve the problem, what is the probability that (i) problem is solved (ii) exactly one of them will solve.

P(A) = ½ , P(B) = 1/3 , P(C) = ¼ , P(A’) = ½ , P(B’) = 2/3 , P(C’) = ¾ ½

(i)1- P(A’B’C’) = 1 – ½ x 2/3 x 3/ 4 = 1- ¼ = ¾ 1

(ii) P(AB”C’) + P(A’BC’) + P(A’B’C) =1/2 x 2/3 x ¾ + ½ x 1/3 x ¾ + ½ x 2/3 x1/4 1
= 11/24 ½

OR

There are 4 cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards . Find the mean value of X.

1½

X:	3	4	5	6	7
P(X) :	1/6	1/6	2/6	1/6	1/6

Mean = 3 x 1/6 + 4x 1/6 + 5 x 2/6 + 6 x 1/6 + 7 x 1/6 = 5 1½

Q28 Evaluate $\int_{-1}^{2}|x^3 - x|dx$

$$\int_{-1}^{0} x^3 - x + \int_{0}^{1} x - x^3 + \int_{1}^{2} x^3 - x$$ 1

$$\left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$ 1

= 11/4 1

OR

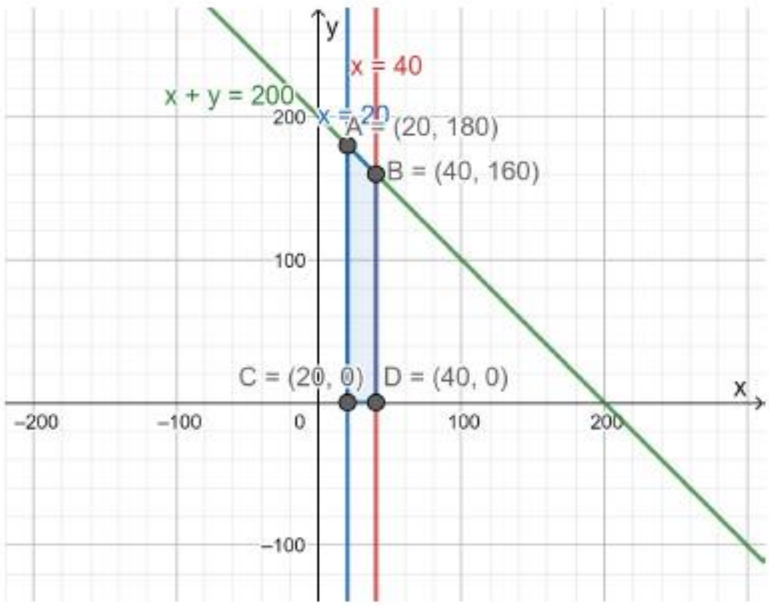
Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{cotx}}$

	$$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}dx$$	½
	$$I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{cosx}}{\sqrt{sinx}+\sqrt{cosx}}dx$$	1
	$$2I=\int_{\pi/6}^{\pi/3}dx$$	1
	$$I=\pi/12$$	½
Q29	Solve the differential equation $x\frac{dy}{dx}sin\left(\frac{y}{x}\right)+x-ysin\left(\frac{y}{x}\right)=0$	
	$$\frac{dy}{dx}=\frac{ysin\left(\frac{y}{x}\right)-x}{xsin\left(\frac{y}{x}\right)}$$	½
	Put y = vx so that $\frac{dy}{dx}=v+x\frac{dv}{dx}$	½
	$$v+x\frac{dv}{dx}=\frac{vsinv-1}{sinv}$$	
	$$x\frac{dv}{dx}=\frac{-1}{sinv}$$	
	$$\int -sinv\ dv=\int\frac{dx}{x}$$	1
	cosv = logx + C	1
	cos(y/x) = logx + C	
	OR	
	Solve the differential equation $(1+x^2)\frac{dy}{dx}$+ y = $\tan^{-1}x$	
	$$\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{\tan^{-1}x}{1+x^2}$$	½
	IF = $e^{\int\frac{1}{1+x^2}}=e^{\tan^{-1}x}$	½
	$y.e^{\tan^{-1}x}=\int\frac{\tan^{-1}x}{1+x^2}.e^{\tan^{-1}x}$	½
	Put t = $\tan^{-1}x$	
	$$y.e^{\tan^{-1}x}=\int te^t dt$$ $$=te^t-e^t$$	1
	$$y.e^{\tan^{-1}x}=e^{\tan^{-1}x}(\tan^{-1}x-1)+C$$	½

Q30 Solve the following Linear Programming Problem graphically:

Maximize Z = 400x + 300y subject to the constraints
x + y ≤ 200 , x ≤ 40 , x ≥ 20 and y ≥ 0.

We have Z= 400x +300y subject to
$x + y \leq 200, x \leq 40, x \geq 20, y \geq 0$
The corner points of the feasible region are C(20,0), D(40,0), B(40,160), A(20,180)



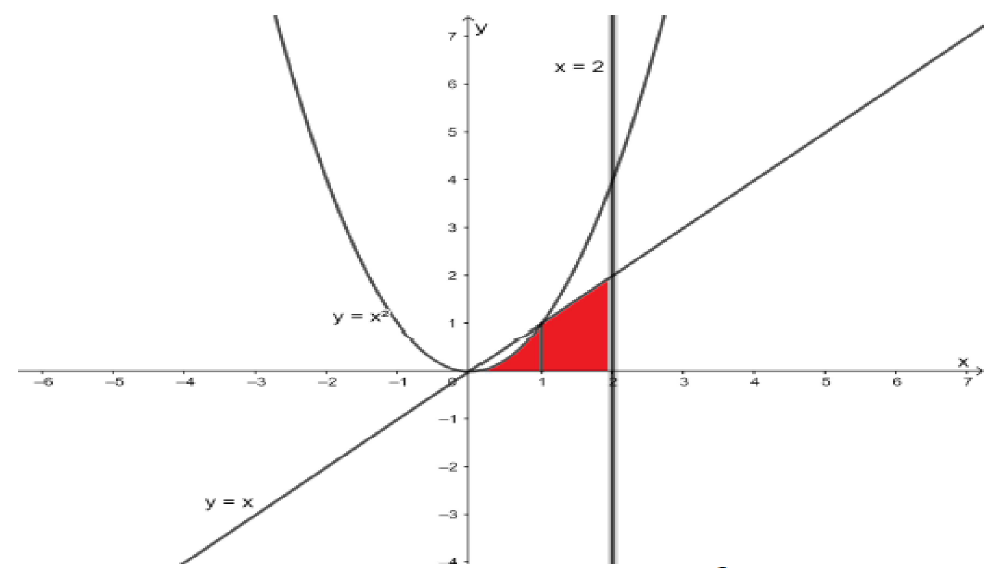
1

Corner Point	**Z = 400x + 300y**
C(20,0)	8000
D(40,0)	16000
B(40,160)	64000
A(20,180)	62000

1

Maximum profit occurs at x= 40, y=160
and the maximum profit =₹ 64,000

1

Q31	Evaluate $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$ Put $e^x = t$ $$I = \int \frac{dt}{\sqrt{-(t^2+4t-5)}}$$ $= \int \frac{dt}{\sqrt{-[(t+2)^2-3^2]}}$ $= \int \frac{dt}{\sqrt{3^2-(t+2)^2}}$ $= \sin^{-1}\frac{t+2}{3}$ $= \sin^{-1}\frac{e^x+2}{3} + C$	½ ½ 1 1
	SECTION D (5 Marks each)	
Q32	Make a rough sketch of the region $\{(x, y): 0 \le y \le x^2, 0 \le y \le x, 0 \le x \le 2\}$ and find the area of the region using integration.  The points of intersection of the parabola $y = x^2$ and the line $y = x$ are (0, 0) and (1, 1). Required Area $= \int_0^1 y_{parabola}\, dx + \int_1^2 y_{line}\, dx$ Required Area $= \int_0^1 x^2\, dx + \int_1^2 x\, dx$ $= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6}$	1 1 2 1
Q33	Let R be the relation defined on the set of natural numbers N as R = $\{(x, y): x \in N, y \in N, 2x + y = 41\}$. Find the domain and range of the relation R. Also verify R is reflexive, symmetric or transitive. Domain = { 1,2 3,………., 20}	1

	Range ={ 1,3,5,7,..........,39} R is not reflexive Eg. (x,x) ∈ $R \rightarrow 3x = 41 \rightarrow x = \frac{41}{3}, not\ a\ natural\ number$ R is not symmetric Let x = 1 and y = 39 2x + y = 41 but 2y + x ≠ 41 R is not transitive (20,1) ∈ R and (1,39) ∈ R but (20,39) does not belong to R **OR** The relation R in the set N x N is defined as follows: For (a, b) and (c, d) ∈ N x N, (a, b) R (c, d) iff ad = bc. Prove that R is an equivalence relation on N x N A relation is equivalence if it reflexive, symmetric and transitive (a,b) R (a,b) →ab = ba which is true, So R is reflexive (a,b)R(c.d) → ad = bc →cb = da →(c,d) R(a,b). So R is symmetric (a,b) R (c,d) and (c,d) R(e,f) → ad = bc and cf = de $\rightarrow adcf = bcde$ $\rightarrow af = be$ $\rightarrow (a,b)R(e,f)$. So R is transitive	1 1 1 1 1 1 1 2
Q34	Find the shortest distance between the lines $\vec{r} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k} + \alpha(\hat{\imath} - 2\hat{\jmath} + \hat{k})$ And $\vec{r} = -\hat{\imath} - \hat{\jmath} - \hat{k} + \beta(7\hat{\imath} - 6\hat{\jmath} + \hat{k})$. a_1=3i+5j+7k, a_2=-i-j-k, b_1 = i-2j+k, b_2= 7i-6j+k a_2-a_1 = -4i-6j-8k b_1xb_2 = 4i+6j+8k (b_1xb_2).(a_2-a_1) = -116 Ib_1xb_2I = √116 Distance = $\left\lvert\frac{(b1xb2).(a2-a1)}{\lvert b1xb2\rvert}\right\rvert$=116/√116 = √116 **OR** Find the vector and Cartesian equations of the line passing through the point (2,1,3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}\ and\ \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.	1 ½ 1 ½ ½ 1 ½

	Equation of the line is r = a + λ b	½
	a = 2i + j + 3k	½
	b = (i+2j+3k) x (-3i+2j+5k) = 4i – 14j +8k	2
	Vector form is r = (2i + j + 3k) + λ(4i – 14j +8k)	1
	Cartesian form is $\frac{x-2}{4} = \frac{y-1}{-14} = \frac{z-3}{8} \rightarrow \frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$	1
Q35	Solve the following system of linear equations using matrix method:	
	x – y +2z = 7, 3x + 4y – 5z = -5 , 2x – y + 3 z = 12	
	A= $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$ X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ B = $\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$	½
	IAI = 4	1
	Minor = $\begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$	1
	Cofactor = $\begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}$	½
	Adjoint = $\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$	½
	A^{-1} = 1/IAI $\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$	½
	X = A^{-1}B = ¼ $\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$	1
	Q36, Q37 and Q38 are Case Based Questions	

Q36	(i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12)	1
	(ii) $f'(x) = -0.2x + m$ Since, 6 is the critical point, $f'(6) = 0 \Rightarrow m = 1.2$	1
	(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$ $f'(x) = -0.2x + 1.2 = -0.2(x - 6)$	

In the Interval	$f'(x)$	**Conclusion**
(0, 6)	+ve	f is strictly increasing in [0, 6]
(6, 12)	-ve	f is strictly decreasing in [6, 12]

2

OR

(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$,
$f'(x) = -0.2x + 1.2, f'(6) = 0$,
$f''(x) = -0.2$
$f''(6) = -0.2 < 0$
Hence, by second derivative test 6 is a point of local maximum. The local maximum value $= f(6) = -0.1 \times 6^2 + 1.2 \times 6 + 98.6 = 102.2$
We have $f(0) = 98.6, f(6) = 102.2, f(12) = 98.6$ — 2
6 is the point of absolute maximum and the absolute maximum value of the function = 102.2.
0 and 12 both are the points of absolute minimum and the absolute minimum value of the function = 98.6.

Q37	(i) Since 'C' is cost of making tank $\therefore C = 70xy + 45 \times 2(2x + 2y)$ $\Rightarrow C = 70xy + 90(2x + 2y)$ $\Rightarrow C = 70xy + 180(x + y)\ [\because 2 \cdot x \cdot y = 8 \Rightarrow y = \frac{8}{2x} \Rightarrow y = \frac{4}{x}\ ]$ $\Rightarrow C = 70x \times \frac{4}{x} + 180\left(x + \frac{4}{x}\right)$ $\Rightarrow C = 280 + 180\left(x + \frac{4}{x}\right)$ (ii) $x \cdot y = 4$ Volume of tank = length × breadth × height (Depth) $8 = x \cdot y \cdot 2$ $\Rightarrow 2xy = 8 \Rightarrow xy = 4$ (iii) For maximum or minimum $\frac{dC}{dx} = 0$ $\frac{d}{dx}(280 + 180(x + \frac{4}{x})) = 0 \Rightarrow 180\left(1 + 4\left(-\frac{1}{x^2}\right)\right) = 0$ $\Rightarrow 180\left(1 - \frac{4}{x^2}\right) = 0 \Rightarrow 1 - \frac{4}{x^2} = 0$ $\Rightarrow \frac{4}{x^2} = 1 \Rightarrow x^2 = 4$ $\Rightarrow x = \pm 2$ $\Rightarrow x = 2$ (length can never be negative)	1 1 2
Q38	(i) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form. $P(E_1) = 0.5$, $P(E_2) = 0.2$, $P(E_3) = 0.3$ $P(\frac{A}{E_1}) = 0.06$, $P(\frac{A}{E_2}) = 0.04$, $P(\frac{A}{E_3}) = 0.03$ Using Bayes' theorem, we have $P(\frac{E_1}{A}) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)}$ $= \frac{0.5 \times 0.06}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03} = \frac{30}{47}$ $\therefore$ Required probability $= P\left(\frac{\bar{E_1}}{A}\right)$ $= 1 - P\left(\frac{E_1}{A}\right) = 1 - \frac{30}{47} = \frac{17}{47}$ (ii) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen the form. $P(E_1) = 0.5$, $P(E_2) = 0.2$, $P(E_3) = 0.3$ $P(\frac{A}{E_1}) = 0.06$, $P(\frac{A}{E_2}) = 0.04$, $P(\frac{A}{E_3}) = 0.03$ $P(A \cap E_2) = P\left(\frac{A}{E_2}\right) . P(E_2)$ $\Rightarrow 0.04 \times 0.2 = 0.008$	2 2